Coherency Analysis in Nonlinear Heterogeneous Power Networks: A Blended Dynamics Approach

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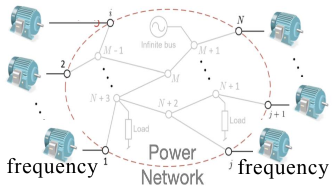
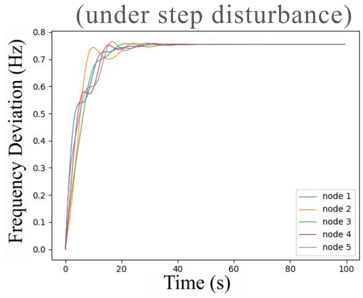
Motivation

Observations: different nodes exhibit similar response to disturbance ("*Coherency*")

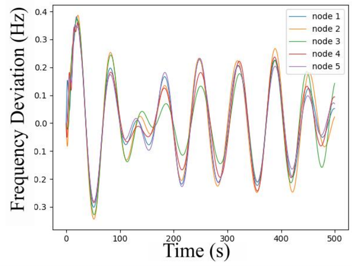


similar frequency responses

(under step disturbance)



(under time-varying disturbance)



Challenges:

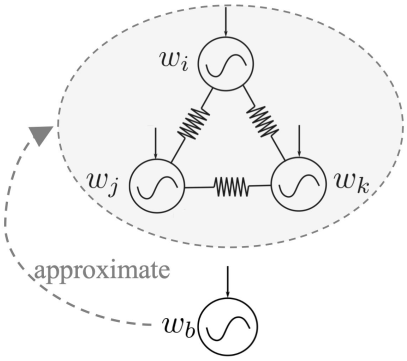
In realistic power networks:

- Heterogeneous & nonlinear dynamics
- Persistent time-varying disturbances

We want to theoretically answer:

- When will nodal responses remain similar (*coherent*)?
- If similar, how to characterize a single trajectory that approximates all responses (*the coherent response*)?

Model & Method



Heterogeneous nodal dynamics (driven by physical laws)

$$M_i\dot{w}_i = -f_i(w_i) + \xi_i - \underbrace{\sum_j B_{ij}\sin(\theta_i - \theta_j)}_{\text{power flow}}$$

frequency; nonlinear damping; disturbance; angle

Blended dynamics (by our construction)

$$\Big(\sum_i M_i\Big)\dot{\hat{w}} = \sum_i\Big(-f_i(\hat{w}) + \xi_i\Big)$$

We use $\hat{w}(t) \in \mathbb{R}$ to approximate $\{w_i(t)\}_{i\in\mathcal{N}}$

Results

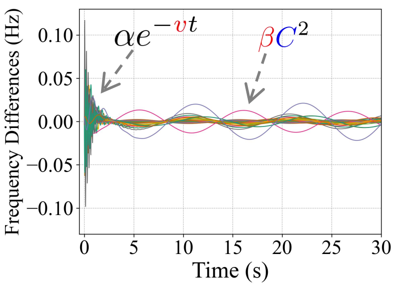


Fig: Evolution of $\hat{w}(t) - w_i(t)$

Assumption:

$\frac{df_i(w_i)}{dw_i}$ is strictly positive and upper bounded

Theorem: (Bounds on approximation error)

$$\max_i |\hat{w}(t) - w_i(t)|^2 \le \alpha e^{-vt} + \beta C^2$$

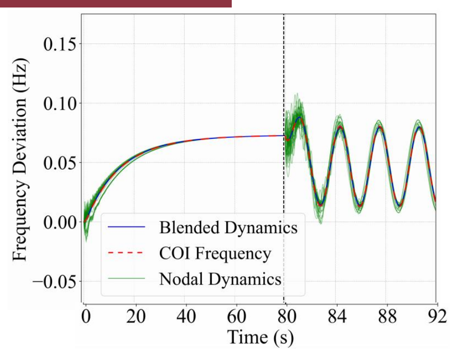
(holds locally under nonlinear power flow)

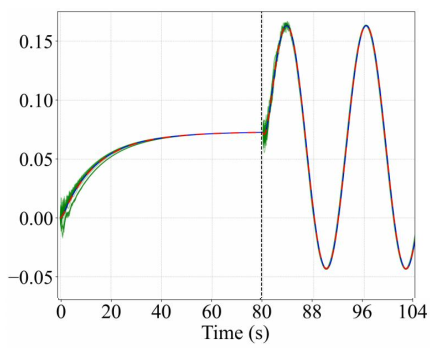
Interpretations:

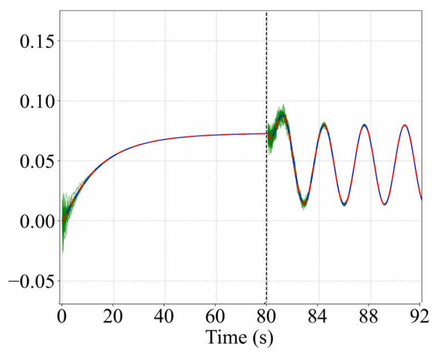
- network with high algebraic connectivity ⟹ large v, small β — ✓ $w_i(t)$ approaches $\hat{w}(t)$ exponentially fast
- disturbances with small time-variation rate ⟹ small C — ✓ $w_i(t)$ follows $\hat{w}(t)$ closely in long term

Blended dynamics is an effective approximation

Experiments







(on Icelandic power grid)

Case 1: Base case

Case 2: Disturbances with larger magnitudes but slower time-variation rates

Case 3: Network with higher connectivity

Reference

[1] Min, H., Pates, R., & Mallada, E. (2025). A frequency domain analysis of slow coherency in networked systems. Automatica, 174, 112184.

[2] Kim, J., Yang, J., Shim, H., Kim, J. S., & Seo, J. H. (2015). Robustness of synchronization of heterogeneous agents by strong coupling and a large number of agents. IEEE Transactions on Automatic Control, 61(10), 3096-3102.